Technical Notes

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Thickness-Induced Lift on a Thin Airfoil in Ground Effect

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Introduction

THERE has been renewed interest recently in the calculation of the aerodynamic characteristics of a wing flying in proximity to the ground. In fact, some vehicles have been designed to take advantage of the positive aspects of ground effect. For the idealized problem of two-dimensional steady potential flow past a flat plate airfoil in the presence of a plane wall, Tomotika et al.[1] and Havelock[2] used conformal mapping to obtain an exact solution. Green[3] obtained a solution for an airfoil of general shape in ground effect. In Refs. 1-3, expansions are also given in chord-to-wall clearance ratio. More recently, accurate numerical solutions using panel methods (surface singularity distributions) are readily obtainable. An early example of this approach is given by Giesing.[4]

In the case where the airfoil disturbance and the chord-to-wall clearance ratio are small, analytical results can be obtained directly using the higher-order thin airfoil theory of Van Dyke.[5] For a wide range of the chord-to-wall clearance ratio, an approach due to Keldysh and Lavrentiev[6] for the corresponding hydrofoil problem is appropriate. Plotkin[7,8] used this method, which involves a series expansion in the chord-to-wall clearance (submergence depth) ratio, to study the first-order effects of thickness, angle of attack, and camber.

In this paper the effect of the thickness of the airfoil on the lift coefficient will be calculated to the second order in the thickness ratio. Analytical results will be presented for the Joukowski airfoil. The first-order angle-of-attack solution for the flat plate will also be given.

Problem Formulation and Method of Solution

The problem under consideration is the steady two-dimensional potential flow of a uniform stream of speed U past a thin symmetrical airfoil of chord $2c$ aligned parallel to the stream. The airfoil is located a distance hc above a plane wall parallel to the stream. All lengths are normalized by c and speeds by U. The coordinate system is shown in Fig. 1. Distances along and normal to the chord are denoted by x and y, respectively, and the origin is located at midchord.

The airfoil is described by the equation $y = \pm\epsilon T(x)$, where ϵ is a small thickness parameter and $T(x)$ is 0(1). The velocity field is characterized by a velocity potential Φ that is normalized by Uc. It satisfies the following mathematical problem:

$$\nabla^2\Phi = 0 \tag{1}$$

$$\Phi_y = 0 \qquad y = -h \tag{2}$$

$$\Phi_y/\Phi_x = \pm\epsilon T'(x) \qquad y = \pm\epsilon T \tag{3}$$

$$\Phi \to x \qquad x^2 + y^2 \to \infty \tag{4}$$

If the disturbance to the stream is modeled by a distribution of sources of strength $\sigma(x)$ per unit length and vortices of strength $\gamma(x)$ per unit length, both normalized by U, along the chordline and if the images of these singularities in the wall are added, the velocity potential becomes

$$\Phi = x + \frac{1}{4\pi}\int_{-1}^{1}\sigma(\xi)\{\log[(x-\xi)^2+y^2] + \log[(x-\xi)^2 + (y+2h)^2]\}d\xi + \frac{1}{2\pi}\int_{-1}^{1}\gamma(\xi) \times\left(\tan^{-1}\frac{y+2h}{x-\xi} - \tan^{-1}\frac{y}{x-\xi}\right)d\xi \tag{5}$$

The only one of Eqs. (1-4) that is not satisfied by Eq. (5) is the body condition (3).

Following Van Dyke,[5] the velocity potential and singularity strengths are expanded in perturbation series in ϵ of the form

$$\begin{aligned}\Phi &= x + \epsilon\Phi_1 + \epsilon^2\Phi_2 + \cdots \\ \sigma &= \epsilon\sigma_1 + \epsilon^2\sigma_2 + \cdots \\ \gamma &= \epsilon\gamma_1 + \epsilon^2\gamma_2 + \cdots\end{aligned} \tag{6}$$

Substitute the expansion for Φ into Eq. (3) and, after the boundary condition is transferred to the chordline, the boundary conditions for Φ_1 and Φ_2 are

$$\Phi_{1y}(x,0\pm) = \pm T'(x) \tag{7}$$

$$\Phi_{2y}(x,0\pm) = \pm[T\Phi_{1x}(x,0\pm)]' \tag{8}$$

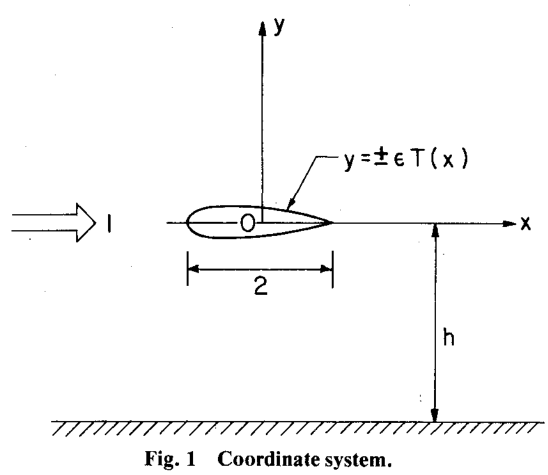

Fig. 1 Coordinate system.

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First-Order Thickness

Consider the first-order problem. The first-order source strength is

$$\sigma_1(x) = 2T'(x) \tag{9}$$

which is the unbounded fluid result. Equation (7) is then an integral equation to determine $\gamma_1(x)$ and can be written

$$\int_{-1}^{1} 2T'(\xi)H(x-\xi)\,\mathrm{d}\xi + \int_{-1}^{1} \gamma_1(\xi)K(x-\xi)\,\mathrm{d}\xi = 0 \tag{10}$$

where

$$H(x) = \frac{-2h}{x^2+4h^2}, \quad K(x) = \frac{1}{x} - \frac{x}{x^2+4h^2} \tag{11}$$

Keldysh and Lavrentiev[6] suggest the following expansion scheme with h^{-1} as the expansion parameter:

$$K(x) = \frac{1}{x} + h^{-1}\sum_{0}^{\infty} K_n\left(\frac{x}{h}\right)^n$$

$$H(x) = h^{-1}\sum_{0}^{\infty} H_n\left(\frac{x}{h}\right)^n \tag{12}$$

$$\gamma_1(x) = \sum_{0}^{\infty} h^{-n}\gamma_{1n}(x)$$

Equations (12) are substituted into Eq. (10) and terms with like powers of h^{-1} are collected. The following system of equations for the unknown $\gamma_{1n}(x)$ is obtained:

$$\int_{-1}^{1} \frac{\gamma_{10}(\xi)}{x-\xi}\,\mathrm{d}\xi = 0$$

$$\int_{-1}^{1} \frac{\gamma_{1n}(\xi)}{x-\xi}\,\mathrm{d}\xi = -\int_{-1}^{1} 2H_{n-1}T'(\xi)(x-\xi)^{n-1}\,\mathrm{d}\xi$$

$$-\int_{-1}^{1} \sum_{m=0}^{n-1} K_m(x-\xi)^m\gamma_{1n-m-1}(\xi)\,\mathrm{d}\xi \qquad n\geq 1$$

$$\equiv f_{1n}(x) \tag{13}$$

The solution is not unique until the Kutta condition at the trailing edge is satisfied. The solution of Eqs. (13) is then

$$\gamma_{1n}(x) = \frac{1}{\pi^2}\left(\frac{1-x}{1+x}\right)^{1/2}\int_{-1}^{1}\left(\frac{1+\xi}{1-\xi}\right)^{1/2}\frac{f_{1n}(\xi)}{\xi-x}\,\mathrm{d}\xi \tag{14}$$

The singular integrals in Eqs. (13) and (14) are to be considered in the Cauchy principal value sense.

The first nonzero value for $\gamma_{1n}(x)$ occurs for $n=3$ and only odd terms are nonzero for $n>3$. Results have been obtained for the first three terms in the series but higher-order terms in h^{-1} can be calculated in a straightforward manner. The thickness function only appears in the $0(\epsilon)$ problem in terms of its moments

$$a_n = \int_{-1}^{1} x^n T(x)\,\mathrm{d}x \tag{15}$$

Second-Order Thickness

The boundary condition for the second-order problem is given in Eq. (8). The second-order source strength is

$$\sigma_2(x) = 2T_2'(x) \tag{16}$$

and the integral equation for $\gamma_2(x)$ is

$$\int_{-1}^{1} 2T_2'(\xi)H(x-\xi)\,\mathrm{d}\xi + \int_{-1}^{1}\gamma_2(\xi)K(x-\xi)\,\mathrm{d}\xi$$

$$= \pi[T(x)\gamma_1(x)]' \tag{17}$$

where

$$T_2(x) = \frac{T(x)}{2\pi}\left[\int_{-1}^{1} 2T'(\xi)\left(\frac{1}{x-\xi} + \frac{x-\xi}{(x-\xi)^2+4h^2}\right)\mathrm{d}\xi\right.$$

$$\left. + \int_{-1}^{1}\gamma_1(\xi)H(x-\xi)\,\mathrm{d}\xi\right] \tag{18}$$

Equation (17) is solved in the same fashion as Eq. (10). However, the first term in the series for $\gamma_2(x)$ cannot be obtained until a particular airfoil geometry is chosen.

First-Order Angle of Attack

To enable one to assess the magnitude of the thickness contribution to the lift of a thin symmetric airfoil in ground effect, the first-order angle-of-attack problem will also be presented. The analysis closely follows that of the first-order thickness problem and will only be outlined here.

Consider a flat plate airfoil described by

$$y = -\alpha x \tag{19}$$

where $\alpha = 0(\epsilon)$. The boundary condition corresponding to Eq. (7) is

$$\Phi_{1y}(x, 0\pm) = -\alpha \tag{20}$$

The disturbance flowfield is modeled by a vortex distribution of strength $\gamma_1(x)$ per unit length and the integral equation for $\gamma_1(x)$ is

$$\int_{-1}^{1}\gamma_1(\xi)K(x-\xi)\,\mathrm{d}\xi = 2\pi\alpha \tag{21}$$

Equation (21) is solved in the same fashion as Eq. (10).

The solution for $n=0$ is the result for an airfoil in an unbounded fluid and is

$$\gamma_{10}(x) = 2\alpha\left(\frac{1-x}{1+x}\right)^{1/2} \tag{22}$$

The first-order vorticity distribution for a symmetrical airfoil at angle of attack in ground effect can be obtained by adding the first-order solutions for thickness and angle of attack. If second-order results are desired, however, care must be taken to distribute the singularity distributions on the chordline, which for nonzero angle of attack no longer coincides with the x axis (see Kennell[9]).

Results and Discussion

The lift coefficient for the airfoil is given by[7]

$$C_L = \int_{-1}^{1}\gamma(x)\,\mathrm{d}x \tag{23}$$

Consider a Joukowski airfoil as an example problem. Van Dyke[5] gives the thickness function for a symmetrical Joukowski airfoil (correct to the second order in ϵ) with thickness ratio 1.30ϵ as

$$T(x) = (1-x)(1-x^2)^{1/2} \tag{24}$$

The first-order result is

$$C_L = \epsilon\left(-\frac{3}{16}h^{-3} + \frac{5}{64}h^{-5} + \frac{279}{4096}h^{-7} + 0(h^{-9})\right) \tag{25}$$

The leading term in the second-order lift coefficient expansion is

$$C_L = -\frac{3}{16}\epsilon^2 h^{-3} \tag{26}$$

The lift coefficient for the first-order angle-of-attack problem is

$$C_L = 2\pi\alpha\left(1 + \frac{1}{4}h^{-2} - \frac{3}{32}h^{-4} - \frac{1}{512}h^{-6} + 0(h^{-8})\right) \tag{27}$$

Note that h^{-1} is the semichord-to-wall clearance ratio.

It is important to note that this problem contains two small parameters, one associated with the airfoil disturbance (ϵ,α) and one associated with the wall disturbance (h^{-1}). Consider the first-order lift coefficient due to angle of attack [Eq. (27)]. The series converges rapidly in h^{-1}. The expansion in h^{-1} of the exact solution of Havelock[2] is

$$C_L = 2\pi\sin\alpha\Big\{1 - \sin\alpha h^{-1} + \frac{1}{4}h^{-2}[1 + 3\sin^2\alpha] - \frac{1}{4}h^{-3}[\sin\alpha + 3\sin^3\alpha] - \frac{3}{32}h^{-4}[3 - 13\sin^2\alpha - 22\sin^4\alpha] + 0(h^{-5})\Big\} \tag{28}$$

The terms linear in α in Eqs. (27) and (28) are identical.

Consider the first-order lift coefficient due to thickness [Eq. (25)]. For a distance from the wall of greater than one chord ($h=2$) better than 1% accuracy is achieved with only one term. Green[3] presents a series expansion for the Joukowski airfoil at angle of attack but includes no terms higher than second order in h^{-1} so that a comparison cannot be made.

Equations (25) and (27) present results linear in ϵ and α and higher order in h^{-1}. The utility of these results depends upon the magnitude of the nonlinear terms that have been neglected. The leading term in the second-order thickness problem is given in Eq. (26) and is of $0(\epsilon^2 h^{-3})$. The leading nonlinear angle-of-attack terms are obtained from Eq. (28) and are of $0(\alpha^3)$ and $0(\alpha^2 h^{-1})$. The leading interaction terms are given in the expansions of Green[3] and are of $0(\epsilon\alpha)$ and $0(\epsilon\alpha h^{-2})$.

For the linear results presented in this paper, the lift force experienced by the airfoil in an infinite fluid ($h\rightarrow\infty$) is decreased by the effect of thickness and increased by the effect of angle of attack.

References

[1] Tomotika, S., Nagamiya, T., and Takenouti, Y., "The Lift on a Flat Plate Placed Near a Plane Wall with Special Reference to the Effect of the Ground Upon the Lift of a Monoplane Aerofoil," Aeronautical Research Institute of Tokyo, Rept. No. 97, 1933.

[2] Havelock, T.H., "The Lift and Moment on a Flat Plate in a Stream of Finite Width," *Proceedings of the Royal Society of London*, Series A, Vol. 166, 1938, pp. 178-196.

[3] Green, A.E., "The Two-Dimensional Aerofoil in a Bounded Stream," *Quarterly Journal of Mathematics*, Vol. 18, 1947, pp. 167-177.

[4] Giesing, J.P., "Potential Flow About Two-Dimensional Airfoils," Douglas Aircraft Company Rept. LB31946, Dec. 1965.

[5] Van Dyke, M.D., "Second-Order Subsonic Airfoil Theory Including Edge Effects," NACA Rept. 1274, 1956.

[6] Keldysh, M.V. and Lavrentiev, M.A., "On the Motion of a Wing Under the Surface of a Heavy Fluid," CAHI, Moscow, 1935; translation in Science Translation Service, STS-75, Cambridge, Mass., 1949.

[7] Plotkin, A., "The Thin-Hydrofoil Thickness Problem Including Leading-Edge Corrections," *Journal of Ship Research*, Vol. 19, No. 2, June 1975, pp. 122-129.

[8] Plotkin, A., "A Note on the Thin-Hydrofoil Theory of Keldysh and Lavrentiev," *Journal of Ship Research*, Vol. 20, No. 2, June 1976, pp. 95-97.

[9] Kennell, C.G., "A Higher Order Theory for the Potential Flow About Thin Hydrofoils," Ph.D. Dissertation, Dept. of Aerospace Engineering, Univ. of Maryland, College Park, Md., Fall 1980.

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Control Laws for Adaptive Wind Tunnels

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Introduction

THE concept of modifying the conditions at wind tunnel walls on a systematic basis to minimize the interference about a model has been pursued by Sears et al.[1-4] and Lo and Kraft.[5] In this Note, a control law is developed for achieving this end. The approach is general and does not depend on the mechanical means used to measure the flow variables (sensors) or to control the flow variables (actuators). Also, unlike previous control schemes, the present one does not require an iterative approach per se. However, it should be noted that Lo and Kraft have developed a so-called "one-step iteration" method for two-dimensional, nonlifting flows using classical subsonic theory. By contrast the present approach is valid for both three-dimensional and lifting flows and, subject to the assumption of linearity, is not dependent on any particular theoretical description of the fluid.

Technical Discussion

Consider a model in a wind tunnel (Fig. 1). Conceptually enclose the model in a control box, two of whose walls coincide with those of the wind tunnel. It is desired to control the flow variables on the walls of the box so that, as far as the flow in the neighborhood of the model is concerned, it is as though the model were in a fluid of infinite extent. The flow outside the box (which exists only in the form of a mathematical model) will be called the exterior flow; the flow inside the box is called the interior flow.

Exterior Flow

From a mathematical model of the exterior flow (the reader may wish to think of this as a classical potential flow model for definiteness, although the discussion is not limited to such a mathematical model), the relationship between normal velocity V_D and pressure P_D on the box wall may be determined. The subscript D is used to denote these as the values on the box wall which obey the desired relationship to simulate a fluid of infinite extent. For V_D and P_D measured at a certain number of discrete points, this relationship may be written in matrix form as

$$\{V_D\} = [E]\{P_D\} \tag{1a}$$

E is an influence coefficient matrix which gives the value of V_D at one point due to a unit P_D at another point. In practice it may be easier to determine the inverse of E, i.e., P_D at one

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